ALGORITHMS TO ENSURE RELIABILITY OF POWER SYSTEM OBSERVABILITY

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1. INTRODUCTION

A notion of topological observability of electric power system (EPS) that determines the existence of solution to the problem of load flow calculation on the basis of measurements was introduced in [1]. The notion is based on a relationship between the rank of observability matrix and its structure which depends on the network topology, set and allocation of measurements. Virtually simultaneously the first algorithms for analysis of topological observability were suggested in [2] and [3].

The algorithm for analysis of topological observability [4, 5] which is made independently for active and reactive EPS models is based on construction of a measurement tree on the network graph. Each branch of the graph is connected with one of measurements available in the network. Active (reactive) model is observable if it corresponds to a spanning tree of measurements in which a node with fixed voltage phase (measured magnitude) is a root node. With several voltage measurements reactive model of EPS will be observable if the measurements on the network graph can be used to construct subsystems of measurement trees with measured voltages at the root nodes that in the aggregate cover all nodes of the network graph. Similar condition provides observability of an active model that contains several measurements of voltage phases.

Should the observability analysis reveal that the measurement tree is disconnected or includes not all nodes of the network graph or subsystems of measurement trees without measured voltage magnitude (phase) are detected, EPS is unobservable. For the EPS to become observable it is necessary to add measurements that make it possible to connect separate subsystems of measurement trees, add the nodes that originally did not belong to the tree, or specify the root nodes with a measured voltage magnitude (phase).

In order to ensure reliability of observability when some measurement devices and remote terminal units (RTU) fail, and when the network topology changes the additional measurements are needed. One of possible solutions to the first problem was given in [6] where the authors suggested adding the redundant measurements in each independent loop and branch of the scheme to provide reliability of observability. An original approach to solution of the second problem with the method of integer linear programming was proposed in [7].

Topicality of the observability study is confirmed by annually published new algorithms for analysis and synthesis of topological, algebraic and nonlinear observability. An increased interest in observability is connected with the emergence of new technology of synchronous vector measurements, which is called Phasor Measurement Units (PMU).

The measurement system on the basis of PMU including measurements of voltage vector at the node of PMU placement and current vectors in the adjacent tie lines is intended to provide complete observability of EPS, and is rather costly. The second structure of PMU measurements that contains measurements of voltage vector and current vector in tie line is most appropriate for practical purposes.

The research aims to develop the algorithms for determining the minimum number of PMUs whose measurements can guarantee EPS observability in the normal operating conditions (Problem 1), when some tie lines are disconnected (Problem 2) and when some PMUs fail (Problem 3). The indicated problems are solved for two types of PMU, with measurements available in EPS and without them.

Abstract

The algorithms for choosing the minimum number of PMUs of two types have been developed. The PMUs are to be placed at nodes and tie lines in power systems to provide topological observability of power systems under normal operating conditions and ensure reliability of topological observability in case of failure of individual tie lines and PMUs, in the absence and presence of conventional measurements. The developed algorithms are implemented in the MATLAB environment. Their efficiency has been proved by computations for a large number of test IEEE schemes and real power systems.
An important study related to the choice of an optimal set of PMUs to maintain observability in the normal operating conditions of EPS and when some tie lines are disconnected is work [8]. The authors of [8] employ the binary search method in combination with the conventional algorithm for analysis of topological observability.

In [9] the choice of the minimum number of PMUs, Problem 1, with and without conventional measurements in EPS is made with the method of integer linear programming. This approach was further developed in [10] which suggested ensuring observability when some PMUs fail, and choosing an optimal set of measurements with account taken of the PMU price. One of the first studies to similarly take into account the price of remote terminal units was [7]. In the absence of conventional measurements in EPS the algorithm [10] allows one to determine the optimal set of PMUs to solve Problems 1 and 3. However, in cases where the conventional measurements of power flow and/or zero injections are available it can result in a solution that contains redundant PMUs.

2. ALGORITHMS FOR CHOOSING AN OPTIMAL SET OF PMUS TO PROVIDE EPS OBSERVABILITY

In order to choose the optimal set of PMUs when solving Problems 1-3 we use the method of integer linear programming. The method implements a simplex algorithm that, on the basis of some basic solution, generates another basic solution that has a better value of objective function as compared to its initial value. This procedure is repeated until the basic solution that meets the optimality conditions is obtained. In the beginning each of the problems is solved for the case without conventional measurements of flows and zero injections, and then – for the case with them.

With conventional measurements available, unlike algorithms in [9-11] that also employ the procedure of integer linear programming, the initial matrices are corrected without changing their size. The rules for a posteriori analysis of the solution obtained are suggested. They make it possible to detect redundant PMUs in the solution, if there are any. The algorithms also allow one to take into account the information on nodes, at which PMU placement is necessary for technical or economic reasons, and the nodes, at which PMU placement is irrational or impossible, for example at dangling or transit nodes.

The choice of an optimal number of PMUs for the case of unavailable conventional measurements and zero injections. Most simply the minimum number of PMUs in all the three problems is chosen on the assumption that observability should be provided only by PMU measurements.

The first type of PMU. In order to choose the minimum number of PMUs that will make EPS observable under normal operating conditions we solve the problem of integer linear programming (Problem 1) [9]

$$\begin{align*}
\min_{x} f^T x \\
A x \geq g
\end{align*}$$

where $A$ – $n \times n$ asymmetrical adjacency matrix that consists of 0 and 1, $n$ – the number of nodes, $f$ and $g$ – the unit vectors, vector of solution $x$ – a binary integer vector, with its elements equal to 0 or 1.

Solution (1) allows observability of all nodes in the calculated scheme of EPS to be provided at least once. Replacement of the unit vector $g$ by the vector whose elements are equal to 2 will provide observability of all nodes in the calculated scheme at least twice (Problem 3). From this condition it follows that when some PMU fails power system will remain observable since measurements of each PMU form a group of noncritical measurements.

In order to choose the minimum number of PMUs which will enable EPS to maintain observability when some tie lines are disconnected it is necessary to solve the integer linear programming problem suggested in [7] (Problem 2)

$$\begin{align*}
\min_{x} f^T x \\
M^T x \geq g
\end{align*}$$

where $M^T$ – the unit vector.
where $M^T$ – an $m \times n$ transposed incidence matrix for undirected graph, $m$ – the number of tie lines, $n$ – the number of nodes in the network graph. Elements of the row of matrix $M^T$, that correspond to the nodes of tie line $i$-$j$, are equal to unity, the remaining elements of the row are equal to zero, $f$ and $g$ – the unit vectors, vector of solution $x$ – a binary integer vector whose elements equal 0 or 1.

Solution (2) provides observability of nodes in the calculated scheme that does not have PMUs at least twice, except for observability of dangling nodes. After disconnection of some tie lines in the electric network observability is not lost and each of measurements entering in PMU is noncritical, except for the current vector measurement in a dangling tie line.

Reliability of observability at switchings is particularly important for distribution networks whose topology is subject to frequent changes. Problem 2 should be solved only for the distribution network part in which backup supply can be used in case of electric tie disconnection. In part of the network being a tree, provision of observability at disconnection of some tie lines makes no sense.

Requirement for one-time control of all tie lines in the electric network which is implied by formulation (2) in the general case can lead to a redundant solution.

In some cases a posteriori analysis of solution (2) makes it possible to sufficiently simply identify redundant PMUs. After a set of PMUs is chosen we determine the number of PMUs required for each node to provide its observability. If observability of node $i$ with PMU and all nodes that have no PMU and are adjacent to node $i$ is provided by two more measurements, the PMU at node $i$ is considered as redundant.

On the other hand there can be situations where solution (2) may turn out to be nonoptimal and it will be impossible to determine redundant PMUs in it, whereas some other solution, obtained, for example, by forced placement of PMU at one of the nodes, will enable one to find an optimal solution. For the forced placement of PMU at node $i$ the columns of matrix $M^T$ that correspond to the nodes adjacent to node $i$ should be zeroed.

A posteriori analysis of solution (2) makes it possible to choose PMUs that should be added to solve Problem 3. Firstly, additional PMUs should be added at all dangling nodes. Secondly, to provide reliability of observability when critical PMU fails causing loss of observability of a node where the PMU is placed, it is necessary to add PMU at a node adjacent to the largest number of nodes with critical PMUs.

The second type of PMU. To choose the minimum number of PMUs of the second type which provide observability of EPS under normal operating conditions (Problem 1), it is necessary to solve the problem [11]

$$\min_{x} f^T x$$

$$M x \geq g$$

(3)

where $M$ – the incidence matrix for undirected graph, the length of unit vectors $f$ and $g$ is equal to the number of tie lines and the number of nodes in the network scheme, respectively.

If elements of vector $g$ in expression (3) are specified equal to 2 it is possible to determine the set of PMUs which will make it possible to maintain observability both when any of the tie lines is disconnected (Problem 2), and when any of PMUs fails (Problem 3). Indeed, it is always possible to allocate measurements so that there will be no doubling measurements of voltage vectors at one and the same node. In this case all measurements will be noncritical, and Problem 2 and Problem 3 will be solved simultaneously. The dangling tie line in Problem 3 should be simulated as a tie with two parallel lines.

The choice of an optimal number of PMUs for the case of available conventional measurements. EPS has a great number of zero injections and conventional measurements of nodal voltage magnitudes, active and reactive power flows and nodal powers. Nevertheless, many power systems are incompletely observable. Observability can be provided by both conventional measurements and PMU measurements.

Let us consider possible approaches to solution of Problems 1-3 for choosing the minimum number of PMUs of the first and second types that should be added to zero injections and power flow measurements to make EPS completely observable. In order to take into account the conventional measurements it is necessary to correct matrix elements in expressions (1)-(3) and/or vector $g$ that correspond to these measurements.

Consideration of power flow measurements. The power flow measurement in tie line $i$-$j$ (Problems 1 and 3) is considered in matrix $A$ by placing unity into row $i$ ($j$) of the adjacency matrix in the columns corresponding to the nodes adjacent to node $j$ ($i$).
In this case the incidence matrix $M$ is corrected by adding unity to the row corresponding to node $i$ ($j$) in the columns corresponding to the tie lines, in which node $j$ ($i$) is one of their nodes.

A kind of integration of nodes $i$ and $j$ into one node takes place, the $i$-th and $j$-th rows of matrices $A$ and $M$ become identical and represent the rows of the adjacency matrix and the incidence matrix for integration of nodes $i$ and $j$.

In Problem 2 the power flow measurement in tie line $i-j$ is taken into account by zeroing the row elements of matrix $M^T$ and the element of vector $g$ that correspond to this tie line.

**Consideration of zero injections.** In Problems 1 and 3 matrices $A$ and $M$ can be formed by using two approaches to consideration of zero injections.

In the first approach the zero injections may be replaced by power flow measurements in one of the tie lines adjacent to zero injections. Such a tie line can be either determined on the basis of analysis of the measurement tree or specified arbitrarily.

If the choice of correspondence between the zero injection and the power flow replacing it is unsuccessful, the global optimum is not always found. Replacement of one tie line associated with the injection by the other tie line can improve the solution. However, the necessity to perform such replacement makes this approach irrational. At the same time if there is a dangling tie line among the tie lines adjacent to the injection, the injection should be put in correspondence only with this tie line.

In the second approach it is assumed that the zero injection at node $i$ provides its observability. This property is taken into consideration by setting the elements of vector $g$, corresponding to the $i$-th rows of matrices $A$ and $M$, equal to zero.

In this case the chosen set of PMUs may turn out to be redundant, which will be revealed in a posteriori analysis of the solution obtained.

For example, when solving Problem 1 for node $i$ with PMU of the first type it is checked whether there are adjacent nodes without PMUs and zero injections, whose observability is provided only by PMU at node $i$. Should there be no such nodes, but there is a zero injection at one of the nodes adjacent to node $i$, whose observability is provided at least by one PMU, such injection is put in correspondence with node $i$ and PMU at this node is considered as redundant.

PMU at node $i$ with zero injection is also redundant, if observability of this node is provided by no less than two PMUs and if it has no more than one adjacent node $j$ without PMU, whose observability is provided only by PMU at node $i$. After removal of the redundant PMU at node $i$ the zero injection of this node is used to provide observability of node $j$.

Some other scenarios for the emergence of redundant PMUs are also analyzed. In all cases they are detected by using the procedure of searching for a tie line that can be placed in correspondence with the zero injection which is similar to the procedure of searching for the maximum matching on the bichromatic graph [2].

In Problem 2 the zero injections are taken into consideration by correction of matrix $M^T$ and the vector $g$. The following situations are possible here:

1. Zero injection at node $i$, that is adjacent to dangling node $j$ is used to provide observability of node $j$, and the row elements of matrix $M^T$ and the element of vector $g$ corresponding to tie line $i-j$ are set to zero.

2. All elements of the column of matrix $M^T$ that corresponds to node $i$ with the zero injection, and the elements of vector $g$ that correspond to tie lines $i-k$ are set to zero, if the degree of node $k$ is higher than or equal to 2. If the degree of nodes $i$ and $k$ is 2 and the injection at node $k$ is not equal to zero, the element of vector $g$ is not zeroed. Removal of the nodes with degree 2 with zero injection from the scheme, as it was done in [8], makes it possible not to check the last condition.

**Application of the software of topological observability analysis to choose an optimal set of measurements for Problem 1.** The procedure of integer linear programming is valid for determination of the optimal set of PMUs in power systems that do not have many conventional measurements or use only PMU measurements.

In a partially unobservable EPS the adjacency matrix can be constructed only for part of the network that does not include the observable nodes connected only with observable nodes. After removal of such observable
nodes and tie lines among the observable nodes it is necessary to take into account the fact that the remaining
nodes in the network are observable. This procedure can essentially reduce the scope of optimization problem
of the number of additional PMUs.

The algorithm of topological observability analysis that makes it possible to determine branches of the
measurement tree, observable and unobservable subsystems for the considered network topology and a set of
conventional measurements can also be used to choose additional PMUs [2]. For this purpose it is necessary to
analyze the subsystems of measurement trees and to choose the nodes allowing integration of the maximum
number of subsystems of trees as the sites for PMU placement. The procedure of checking observability for the
analysis of corrected measurement tree should be repeated, after each regular PMU is chosen. In order to ana-
lyze topological observability in this case it is necessary to sort out the conventional measurements and leave
only those at which the sets of nodes of incompletely observable subsystems of the active and reactive models
coincide. The term "incompletely observable" to a greater extent concerns observability of the active model
because there is no voltage phase measurement in subsystem of the kind. The reactive model in this case can
be both observable and incompletely observable which depends on the location of voltage magnitude measure-
ments in it. Further, after each additional PMU is chosen, the procedure of observability analysis can be per-
formed on the basis of the software intended for topological observability analysis of the reactive model [4].

3. CASE STUDY

Fig. 1 presents a 43-node HV distribution network, for which availability of both zero injections and power
flow measurements is simulated. The results of choosing the optimal sets of PMUs of the first and second types
when solving Problems 1-3 are given in Table 1. All the calculations were carried out by using the software de-
veloped in the MATLAB environment. The solutions obtained were tested for availability of missing and redun-
dant measurements on the basis of the software [4].

Fig. 2 illustrates the possibility of choosing the optimal set of PMUs that coincides with the set determined
in line 2 of Table 1 (Problem 1 in terms of power flow measurements) for the network with removed observable
nodes that are connected only with observable nodes. Grey boxes represent the remaining observable nodes
without PMUs; grey and colorless circles — observable and unobservable nodes with PMUs installed.

While placement of conventional power flow measurements at the beginning or end of tie lines makes no
difference when solving Problems 1 and 2, the situation when power flow measurements belong to one and the
same RTU can substantially influence the optimal solution to Problem 3. This is due to the fact that the choice of the optimal PMU set requires that observability be provided both at failure of any PMU and at failure of any RTU. Since the adjacency matrix $A$ does not take into account whether the power flow measurements belong to RTU, the solution to Problem 3 should be additionally analyzed when individual PMUs and individual RTUs fail, for example, by using the software of topological observability analysis.

The optimal set of PMUs of the 1st type that is presented in line 10 of Table 1 was obtained on the assumption that the power flow measurements belong to RTUs as follows: RTU 1 — 13-14, 13-24,13-38; RTU 2 — 15-32; RTU 3 — 16-13, 16-15, 16-23, 16-31, 16-39; RTU 4 — 40-29; RTU 5 — 41-40; 41-42, RTU 6 — 43-20, 43-22; RTU 7 — 42-43; RTU 8 — 23-41. If to integrate the measurements of RTU 5 and RTU 8, RTU 6 and RTU 7 it will be necessary to add PMUs at nodes 7 and 8 to the set of PMUs shown in line 10 of Table 1.

At the same time the set of PMUs shown in line 9 of Table 1 corresponds to the integration of RTU 5 and RTU 8, RTU 6 and RTU 7. For PMU of the 2nd type Problem 3 for the case of available conventional measurements was not solved.

<table>
<thead>
<tr>
<th>Problems</th>
<th>1st type of PMUs – node numbers (2nd type of PMUs – tie line numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Problem 1, without power flow measurements and zero injections</td>
<td>1, 13, 16, 17, 19–27, 32, 41 – 15 PMU (1, 4–6, 9–13, 10–13, 19, 21, 23, 25–27, 30, 33, 37, 42–44, 46, 47 – 23 PMU)</td>
</tr>
<tr>
<td>2. Problem 1, with power flow measurements</td>
<td>1, 17, 19, 20–27, 32 – 12 PMU (1, 4–6, 9–13, 27, 31, 33, 35, 37, 43, 44 – 16 PMU)</td>
</tr>
<tr>
<td>3. Problem 1, with power flow measurements and zero injections</td>
<td>1, 5, 11, 18, 26, 28, 34 – 7 PMU (1, 6, 12, 14, 15, 21, 27, 31, 35, 3, 44 – 11 PMU)</td>
</tr>
<tr>
<td>5. Problem 2, with power flow measurements</td>
<td>1, 6, 12, 17–27, 31, 32, 34 – 17 PMU (1–3, 6–8, 13–15, 26–32, 36–41, 44 – 23 PMU)</td>
</tr>
<tr>
<td>6. Problem 2, with power flow measurements and zero injections</td>
<td>1, 5, 12, 17–19, 21, 25, 26, 31, 34 – 11 PMU (1–15, 27–33, 36, 37, 39, 41, 43, 44 – 28 PMU)</td>
</tr>
<tr>
<td>8. Problem 3, with power flow measurements</td>
<td>1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 35, 36, 37, 38, 39 – 28 PMU</td>
</tr>
<tr>
<td>9. Problem 3, with power flow measurements and zero injections</td>
<td>1, 2, 6, 12, 18, 24, 25, 27–29, 32, 34, 35, 38, 39 – 15 PMU</td>
</tr>
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</table>

4. CONCLUSION

The algorithms and the MATLAB software have been developed for choosing the optimal number of PMUs to provide topological observability in normal conditions of EPS operation and reliability of EPS observability at failure of individual tie lines and individual PMUs for the cases with and without conventional measurements. The software has been tested on a great number of schemes and real EPSs.
REFERENCES


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